

# Plasma suppression of beam-beam interaction in a muon collider

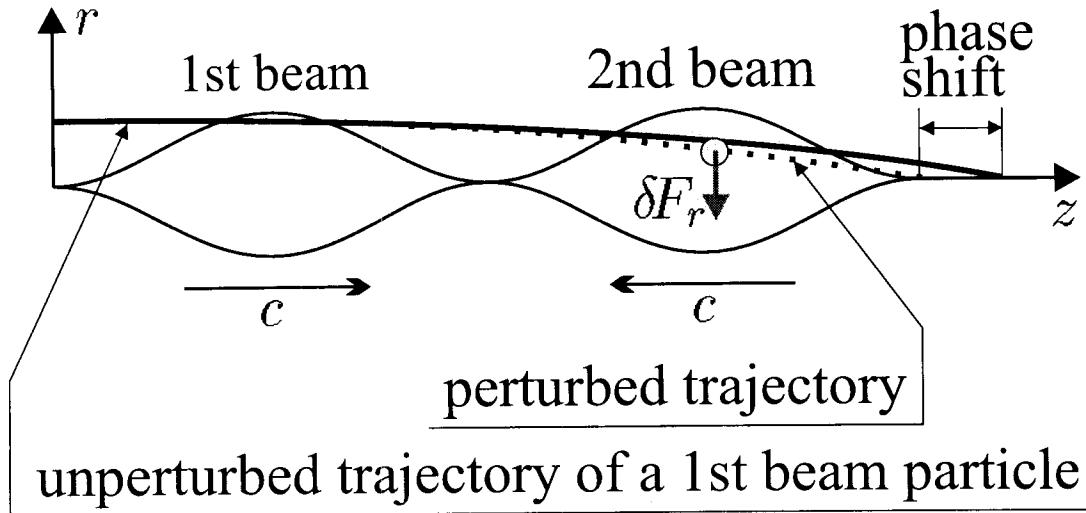
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- 
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## Why plasma compensation ?

Electromagnetic interaction of the beams in the collision region can cause an instability → limits the luminosity



Measure of beam-beam interaction --

$$\text{tune-shift parameter } \xi_0 = \frac{Nr_\mu}{4\pi\varepsilon_n} \quad (\text{round beams})$$

$\xi < \xi_{max} \sim 0.1$  for a muon collider

( $\varepsilon_n$  is the normalized transverse emittance,  $N$  particles in each beam,  $f$  collisions per unit time, one interaction point with beta function  $\beta_c$ ,  $\gamma_b$  is the relativistic factor of the beams,  $r_\mu$  is the classical muon radius,  $L$  is the luminosity for collisions in vacuum)

If  $\xi_{max}$  is limited, then

$$L = \frac{N^2 f}{4\pi\sigma_r^2} < \frac{N f \gamma_b}{r_\mu \beta_c} \xi_{max}, \quad \varepsilon_n > \frac{Nr_\mu}{4\pi\xi_{max}}.$$

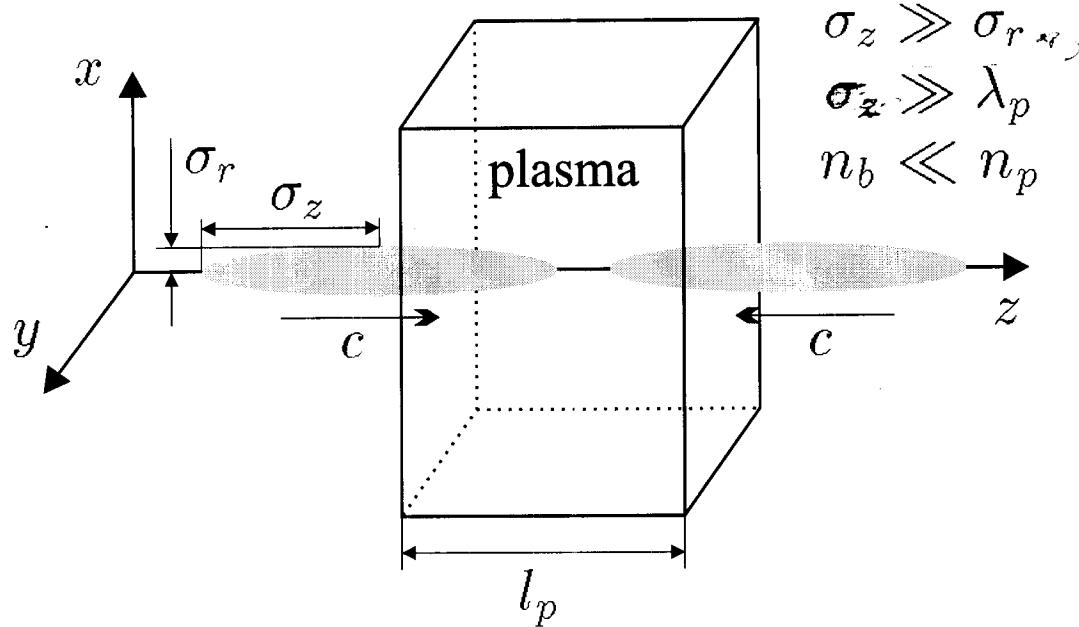
Plasma compensates the fields of the beams

→  $N/\varepsilon_n$  can be increased with no instability

→ higher luminosity.

## What is plasma compensation

The beams induce charges and currents in plasma, which locally compensates the charge and current of the beams:



“Bad” features of the plasma are possibly not so bad for a muon collider:

- ! no reduction of beam lifetime (short-lived  $\mu$ ),
- ! beam density  $\ll$  electron density in solids (lithium jet),
- ? background.

## Plasma response: linear approximation

Basic equations

$$\text{rot } \vec{B} = -4\pi e \left( n \frac{\vec{v}}{c} + n_{b1} \vec{e}_z + n_{b2} \vec{e}_z \right) + \frac{1}{c} \frac{\partial \vec{E}}{\partial t},$$

$$\text{rot } \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad \frac{\partial n}{\partial t} + \text{div}(n \vec{v}) = 0,$$

$$\frac{\partial \gamma \vec{v}}{\partial t} + (\vec{v} \nabla) \gamma \vec{v} = -\frac{e}{m} \vec{E} - \frac{e}{mc} [\vec{v} \times \vec{B}].$$

in case of low density beams can be linearized, e.g.,

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \vec{E}.$$

Linear plasma response was studied by

**D. H. Whittum, A. M. Sessler, J. J. Stewart, and S. S. Yu** Part. Accel., v. 34 (1990), p. 89.

**A. M. Sessler and D. H. Whittum** AIP Conf. Proc., v. 279, p. 939.

**G. V. Stupakov and P. Chen** Phys. Rev. Lett., v. 76 (1996), p. 3715.

It was found that the good compensation requires

$$n_p \gg n_b \quad \text{and} \quad k_p \sigma_r \gg 1$$

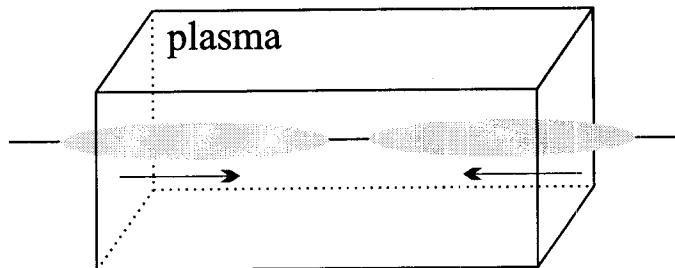
where  $k_p = \omega_p/c$ ,  $\omega_p = \sqrt{4\pi n_p e^2 / m_e}$  is the plasma electron frequency.

Then the compensation degree is  $\xi/\xi_0 \sim \frac{1}{(k_p \sigma_r)^2}$ .

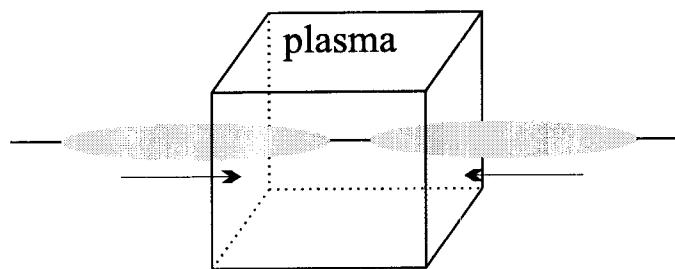
## The role of plasma thickness

Plasma focuses the beams (plasma lens effect)

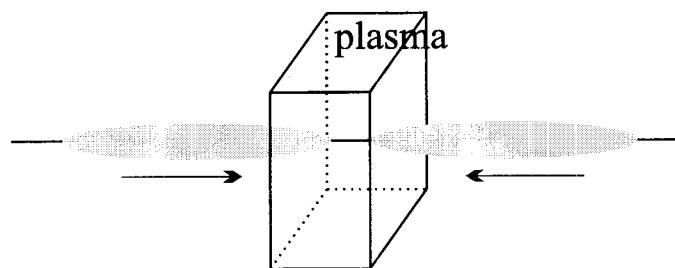
→ at certain plasma thickness the beams get the minimal additional radial momentum.



beams are overfocused by the plasma



minimum transverse momentum gained,  
 $l_p = l_{min}$



beams meet mostly in vacuum, no compensation

For the linear plasma response     $l_{min} = \sigma_z \sqrt{2 \ln(k_p^2 \sigma_r^2 - 1)}$ ,

$$\xi/\xi_0 = \frac{1}{4} \left( \operatorname{erf} \left( \frac{l_p}{\sqrt{2}\sigma_z} \right) + \sqrt{\frac{2}{\pi}} \cdot \frac{l_p}{\sigma_z} \right) \int_0^\infty \frac{x^3 e^{-x^2/4} dx}{x^2 + 2k_p^2 \sigma_r^2} + \\ + \left( 1 - \operatorname{erf} \left( \frac{l_p}{\sqrt{2}\sigma_z} \right) \right),$$

$$\left( \frac{\xi}{\xi_0} \right)_{l_p=l_{min}} \approx \frac{1}{k_p^2 \sigma_r^2} \left( 1 + \frac{8 \ln(k_p^2 \sigma_r^2 - 1) + 1}{4 \sqrt{\pi \ln(k_p^2 \sigma_r^2 - 1)}} \right).$$

## Plasma response: nonlinear approximation

Linear model is valid if  $n_b \ll \frac{n_p}{(k_p\sigma_z)^2}$  — not the case in colliders.

More accurate approach (valid for  $k_p\sigma_z \gg \sqrt{n_p/n_b}$ ):  $\frac{\partial}{\partial t} = 0$ ,

$$\text{rot } \vec{B} = -4\pi e \left( n \frac{\vec{v}}{c} + n_{b1} \vec{e}_z + n_{b2} \vec{e}_z \right), \quad \vec{B} = \frac{m_e c}{e} \text{rot } \gamma \vec{v},$$

$$\text{div } \vec{E} = 4\pi e(n_p + n_{b1} - n_{b2} - n), \quad \vec{E} = -\frac{1}{c} [\vec{v} \times \vec{B}].$$

**D. H. Whittum** Phys. Fluids B, v. 4 (1992), p. 476.

**K. V. Lotov** Phys. Plasmas, 1996, v. 3, p. 2753.

**I. A. Kotelnikov, V. N. Khudik** Plasma Physics Reports, v. 23 (1997), p. 130.

**V. N. Khudik and K. V. Lotov** Plasma Physics Reports, v. 25 (1999), p. 149.

- No analytic solution to equations.
- Correct for  $n_b \gtrsim n_p$  (also describes ion channel formation).
- Validates the use of the linear approximation for

$$\frac{n_p}{(k_p\sigma_z)^2} \lesssim n_b \ll n_p.$$

- Gives the correct  $E_r$  (important for plasma ion dynamics).
- Should be used if  $N \gtrsim \sigma_z/r_e$ .  
(Otherwise the linear approximation can be used, it is violated at plasma densities at which the compensation already disappears).

## Numerical example

### Parameter sets for simulations

Parameter	Value
Beam energy, $W$	5 TeV
Number of particles in each beam, $N$	$5 \cdot 10^{12}$
Beam length, $\sigma_z$	1 cm
Beam radius, $\sigma_r$	$0.6 \mu\text{m}$
Beam density, $n_b$	$9 \cdot 10^{19} \text{ cm}^{-3}$
Relativistic factor, $\gamma_b$	$5 \cdot 10^4$
Normalized emittance, $\varepsilon_n$	1.7 mm mrad
Number of turns	1400
Tune-shift parameter without a plasma, $\xi_0$	3.2
Optimum thickness (lithium), $l_{min}$	3.6 cm
Expected tune-shift parameter in lithium, $\xi$	0.02

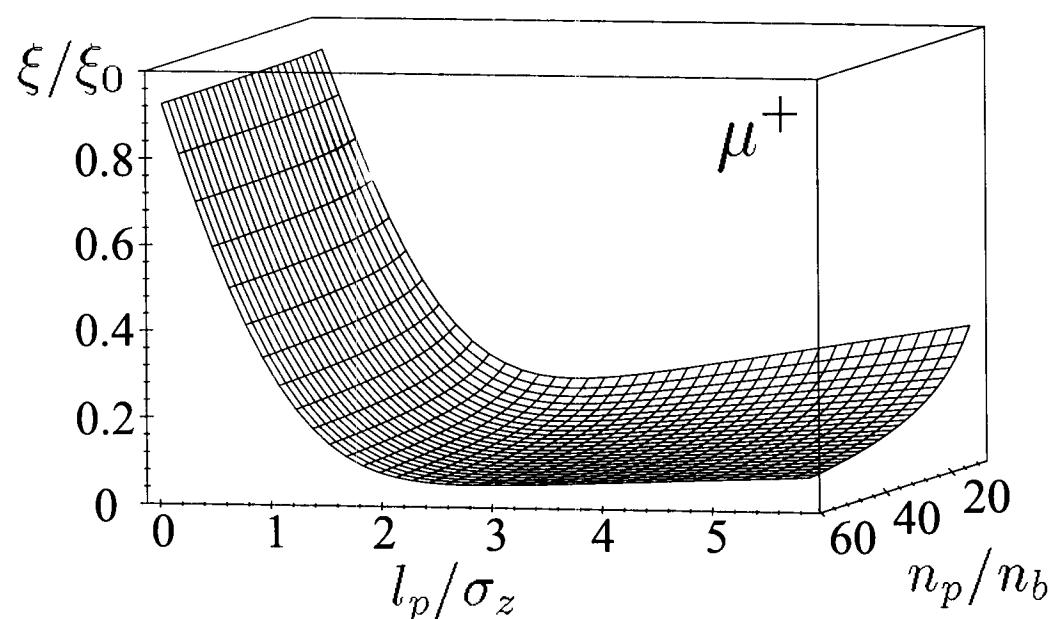
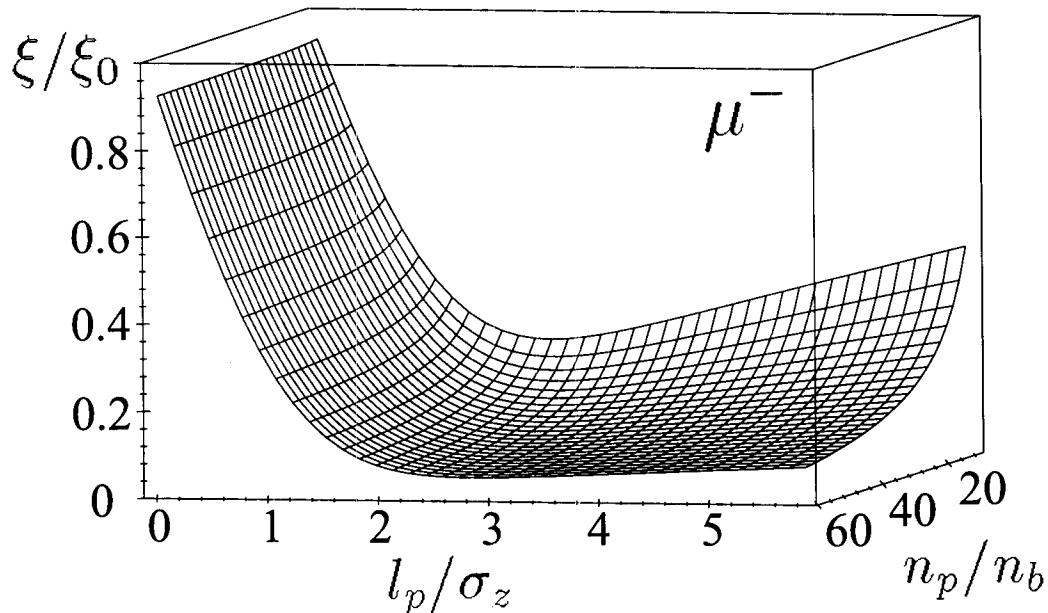
Plasma density,  $n_p$

$5 \cdot 10^{22} \text{ cm}^{-3}$

$c/\omega_p = 0.023 \mu\text{m}$

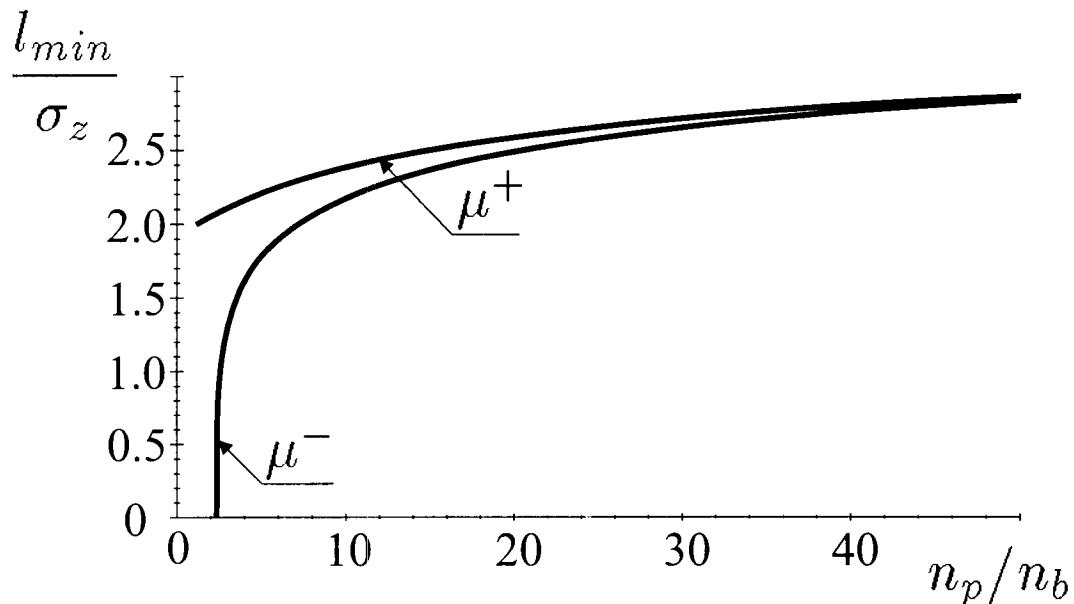
$k_p b_2 = 25$

Dependence of the compensation degree on the density  
and thickness of the plasma  
(nonlinear approximation)



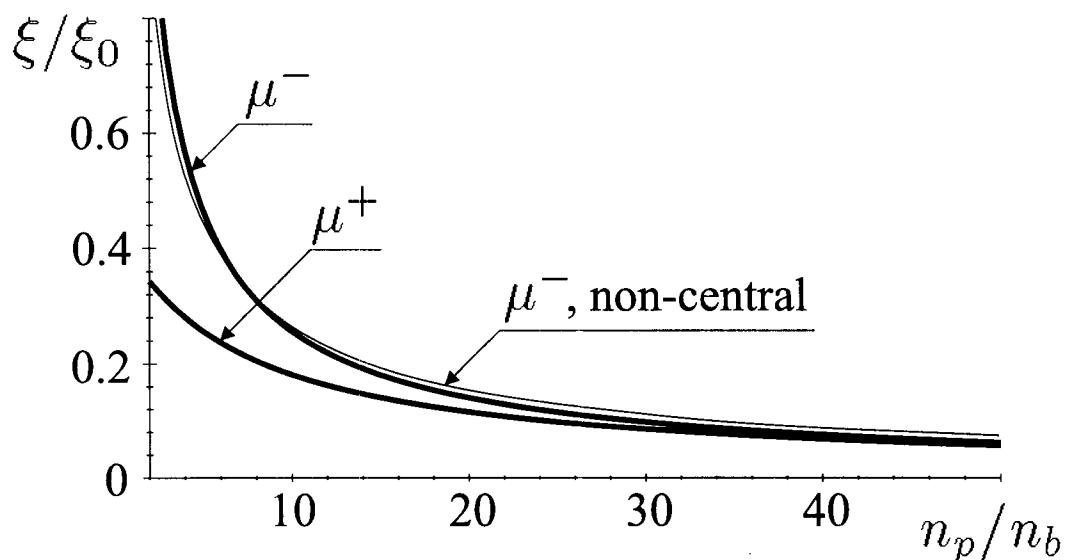
Compensation rates for  $\mu^+$  and  $\mu^-$  are different because  
of nonlinear effects.

Plasma thickness ensuring the best compensation



The function obtained from the linear theory nearly coincides with the optimum thickness for  $\mu^-$  beam.

Compensation degree for the optimum plasma thickness.

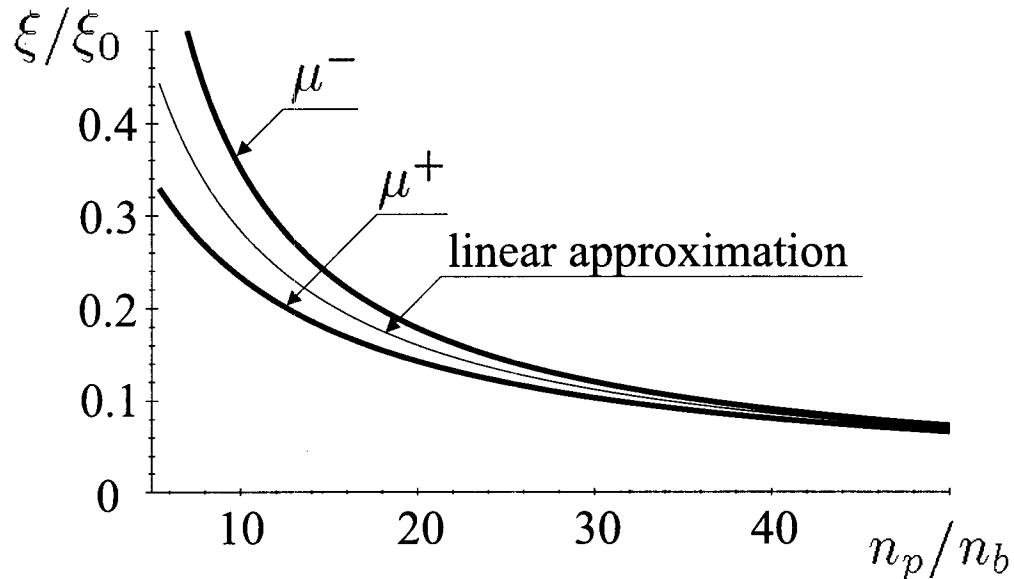


The compensation at non-central cross-sections of the beam ( $\pm\sigma_z$  in longitudinal direction) is almost as good as at the beam center.

## The role of the radial electric field $E_r$

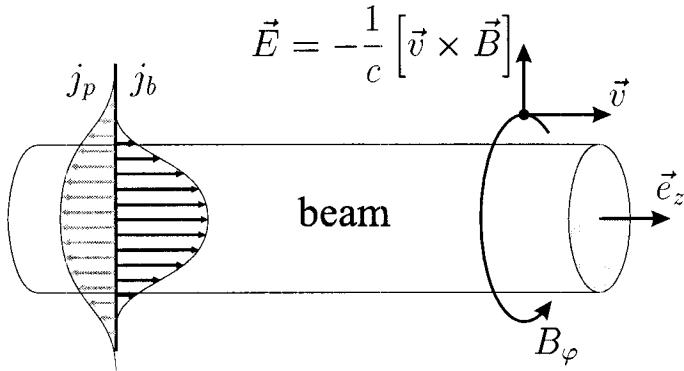
- Introduces a difference in compensation of  $\mu^+$  and  $\mu^-$ .

Compensation degree at a fixed plasma thickness ( $l_p = 4\sigma_z$ ).



$$\text{Relative difference} \sim \frac{E_r}{B_\varphi} \sim \frac{v_z}{c} \sim \frac{n_b}{n_p} \ll 1.$$

- Always pushes the ions from the beams's region (!).



plasma current is directed opposite to the beam's current, but has a smaller value

## Ion motion

Important if  $\sigma_z \gtrsim l_i \sim c\sqrt{\frac{M_i \sigma_r}{e E_r}}$

$$E_r \sim \frac{v_z B}{c}, \quad B \sim \frac{B_{vac}}{(k_p \sigma_r)^2}, \quad B_{vac} \sim e n_b \sigma_r, \quad v_z \sim c \frac{n_b}{n_p}$$

Ions are immobile if  $\sigma_z \ll l_i \sim \sigma_r \frac{n_p}{n_b} \sqrt{\frac{M_i}{m_e}}$

For chosen parameters  $\sigma_z \sim 0.15 l_i$  — not much.

## Beam filamentation

Like currents attract  $\rightarrow$  filamentation (Weibel instability)

R vs. Z

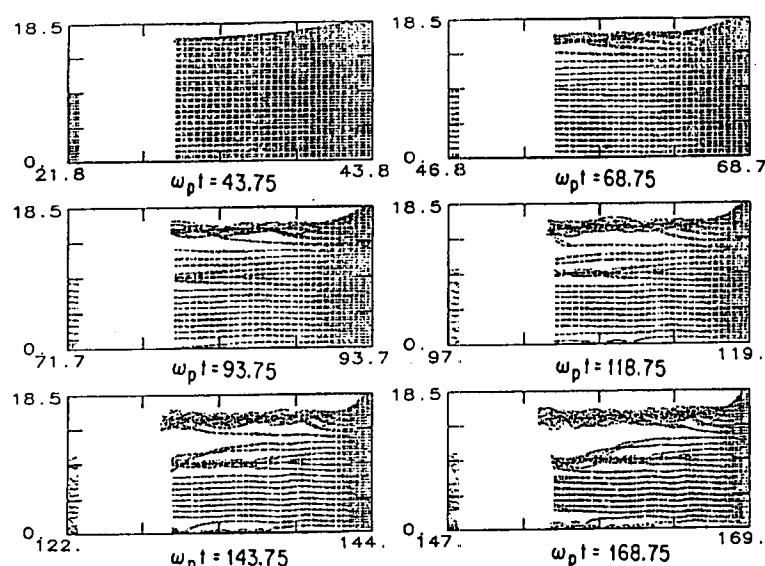


Fig. 5. The time development of monoenergetic driving and accelerated bunches in real space ( $\omega_p t = 43.75, 68.75, 93.75, 118.75, 143.75$ , and  $168.75$ ). The filaments proceed to coalesce into large filaments.

single beam:

growth rate

$$\Gamma_W \sim \sqrt{\frac{m_e n_b}{\gamma_b m_\mu n_p}} \omega_p$$

thermal stabilization if

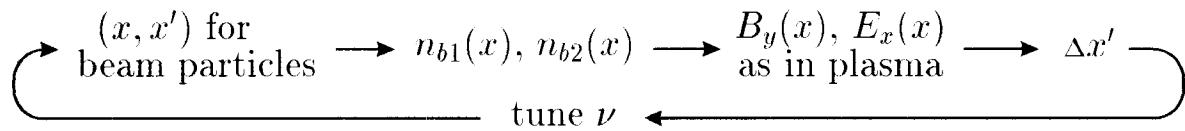
$$v_\perp/c > \Gamma_W/\omega_p$$

Here  $\frac{\sigma_z \Gamma_W}{c} \sim 5.6$ .

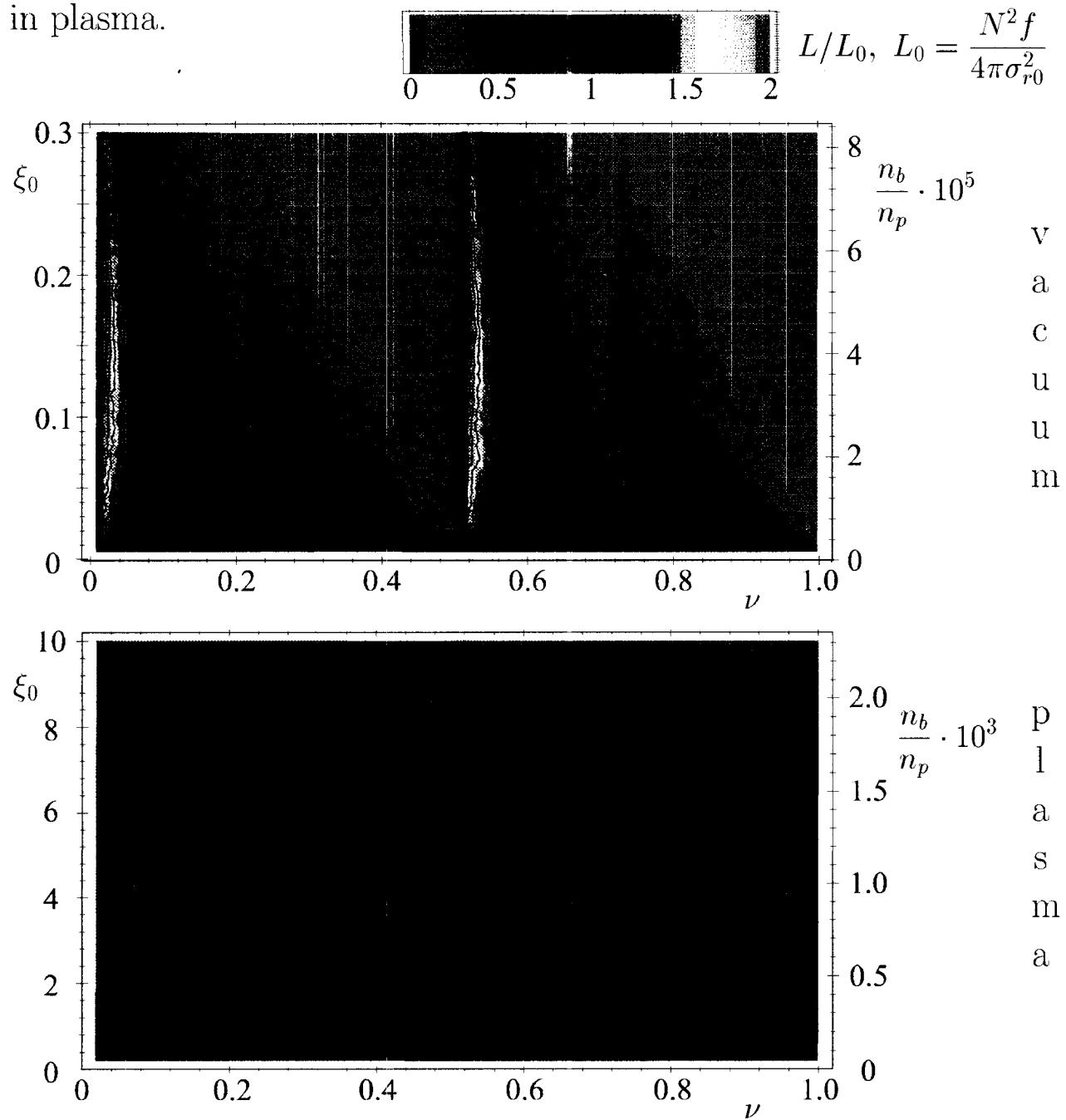
$$\frac{v_\perp \omega_p}{c \Gamma_W} \sim 4.5$$

— not much

## Simulation of beam-beam interaction



1d (plane geometry), plasma is electron fluid, beams are macroparticles, 1400 beam-beam interactions (instant push) in vacuum or in plasma.



Stability threshold  $\ll \xi_{max,vac} \cdot (k_p\sigma_r)^2 \sim 60$ . (!!!)

# Multiple ionization of lithium

	$\text{Li}^+ \rightarrow \text{Li}^{++}$	$\text{Li}^{++} \rightarrow \text{Li}^{+++}$
max. cross-section, $\sigma_i, \text{ cm}^{-3}$	$4 \cdot 10^{-18}$	$10^{-18}$
at $e^-$ energy	300 eV	300 eV
or $v/c \approx$	0.025	0.025

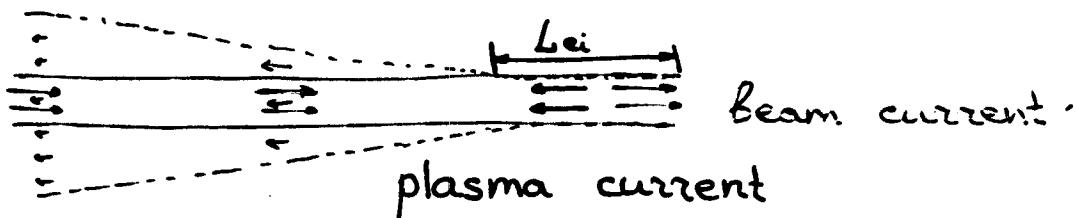
$$\text{Lion}^{++ \rightarrow +++} \sim \frac{c}{n_p \sigma_i v} \sim 10^{-3} \text{ cm} \Rightarrow \text{lithium is fully ionized}$$

## Collisions in plasma (thanks to Valery Telnov)

$$\lambda_{ei} = \frac{v}{\gamma_{ei}} = \frac{m_e^2 v^4}{4\pi Z^2 \Lambda e^4 n_i} \sim 0.2 \text{ cm} \cdot \frac{v^4}{c^4}$$

↑ in lithium ( $Z=3$ )

Causes diffuse widening of plasma current  
(not stopping!)



$$L_{ei} \sim \lambda_{ei} \frac{\sigma_2 \omega_p}{c^3} \quad \text{requires} \quad K_p \sigma_2 \gg 1$$

$\underbrace{v \sim 0.3 c}_{\text{in plasma}}$

$$I_B \gg 1 \text{ kA} \quad \text{or} \quad \sigma_2 \ll N' \omega_p$$

## Conclusion: further steps for theorists

- + Various models of plasma response are developed.
- + First notion of plasma capabilities is obtained.
- ? Background.
- ? Transverse beam stability.
- ? Interplay of beam-plasma and beam-beam instabilities.
- ? Multiple lithium ionization.
- ? Ion dynamics.
- ? 2d and 3d simulations.
- ? Diffusion of the plasma current